

1. Tim the Tarantula King decides to go on a voyage through the Amazon Rainforest to find the perfect meal. To start his journey, he decides to make a sailboat. Which of these sails has the largest area?

The area of the first triangle is  $3 \cdot 4 \cdot \frac{1}{2} = 6$ . The area of the square is  $3 \cdot 3 = 9$ . The area of the equilateral triangle is  $\frac{4^2 \cdot \sqrt{3}}{4} = 4\sqrt{3}$ . This is approximately equal to 7. The area of the circle is  $2^2 \cdot \pi = 4\pi$ , which is approximately equal to 12.  $\boxed{D, \text{ a circle with a radius of } 2}$ .

2. Before setting out, Tim asks his friend Linsey the Lizard to come with him. However, wherever Linsey goes, she needs to bring her favorite log, which is the shape of a cylinder with a radius of 2 cm and height of 8 cm. What is the volume of the log in  $\text{cm}^3$ ?

The volume of the cylinder is  $2^2 \cdot \pi \cdot 8 = \boxed{A, 32\pi}$ .

3. Tim and Linsey set sail! On their journey, they encounter Yimo the capybara collecting seeds. If Yimo wants to collect at least  $192\pi \text{ cm}^3$  of seeds, and each seed is in the shape of sphere with radius 2 cm, how many seeds must Yimo collect?

The volume of each seed is  $\frac{4}{3} \cdot 2^3 \cdot \pi = \frac{32\pi}{3}$ . We now have  $\frac{192\pi}{32\pi/3} = \boxed{D, 18}$ .

4. Linsey, seeing the seeds, becomes hungry and steals some. Yimo sees her running away with his seeds and gets angry. While Tim and Linsey escape on their boat, Yimo decides to fire seeds at them in hopes of sinking their boat. Yimo is at the point  $(2, 3)$  while Tim and Linsey are at the point  $(7, 9)$ . How far does Yimo have to fire the seeds in order to reach Tim and Linsey?

Using the distance formula we have  $\sqrt{(7-2)^2 + (9-3)^2} = \sqrt{25 + 36} = \boxed{B, \sqrt{61}}$ .

5. Tim and Linsey manage to escape! While continuing their journey, they meet David, a poison dart frog. David tells them there is a stream along the line  $y = -2$  that contains honey. Tim, at the point  $(0, 7)$  decides that he will get some honey and tells Linsey to meet him at the point  $(24, 7)$ . What is the minimum distance that Tim must travel to get honey and then meet Linsey?

Reflect  $(24, 7)$  over the line  $y = -2$  to get  $(24, -11)$ . Then from distance formula, the distance between this point and  $(0, 7)$  is  $\sqrt{24^2 + 18^2} = \boxed{B, 30}$ .

6. When Tim sees the honey stream, he encounters Queen Bee Nonoko, who decides that she will give Tim honey only if he can find the outer surface area of her hive. If her hive is made exclusively of 156 regular hexagons with side lengths of 6, what should Tim say in order to get some honey?

The area of each hexagon is  $\frac{3}{2} \cdot 6^2 \cdot \sqrt{3} = 54\sqrt{3}$ . Thus, the outer surface area is  $156 \cdot 54\sqrt{3} = \boxed{A, 8424\sqrt{3}}$ .

7. Tim got it right and gets some honey! As a parting gift, Nonoko decides to give Tim a hat in the shape of a frustum. The hat has a top radius of length 3, a bottom radius of length 6, and a height of 4. What is the surface area of the hat?

We must first find the slant height to be  $\sqrt{(6-3)^2 + 4^2} = 5$ . We now have that the total surface area is  $3^2\pi + 6^2\pi + 5 \cdot (3+6)\pi = \boxed{D, 90\pi}$ .

8. After meeting back up with Linsey, Tim continues on his journey. After a while, Tim sees a stowaway: a Nematode (a super big nematode)! In exchange for safe transit, the Nematode offers to give Tim a flower in the shape of a 22-sided  $n$ -gon. How many diagonals does this flower have?

Using the formula for finding the number of diagonals, we have  $\frac{22(22-3)}{2} = 19 \cdot 11 = \boxed{B, 209}$ .

9. Tim wants to know how the sum of the exterior angles of the flower. Find the sum of the exterior angles of a 22-sided polygon.

The sum of the exterior angles of every polygon is  $\boxed{C, 360}$ .

10. Linsey, realizing that the Nimatode has been eating her seeds, kicks the Nimatode into the river. This leaves a puddle made of 2 concentric circles of radii 8 and 12. What is the area between the circles?

We can find the area of the circles and then subtract. We have  $12^2\pi - 8^2\pi = 144\pi - 64\pi = \boxed{E, 80\pi}$ .

11. As they continue along their voyage, Tim and Linsey see a sign that reads “Aisle of Monkeys” and decide to get off the boat and travel through the aisle. While walking, they see Aaron the Squirrel Monkey, climbing across the trees on a path modelled by the line  $y = 8x + 4$ . In order to avoid Aaron, Tim and Linsey travel along the path  $y = 8x + 9$ . What is the distance between the two paths?

The distance between the two lines is the same as doing the distance from a line to a point on the other line. We can note that  $(0, 4)$  is on the first line. Thus, we now use the formula for finding the distance from a line to a point:  $\frac{8 \cdot (0) + (-1) \cdot (4) + 9}{\sqrt{8^2 + (-1)^2}} = \frac{5}{\sqrt{65}}$ . Multiplying the top and bottom by  $\sqrt{65}$  gives us  $\frac{5\sqrt{65}}{65} = \boxed{D, \frac{\sqrt{65}}{13}}$ .

12. Moving onward, Tim and Linsey spot Shaoyang the Howler Monkey. Noticing them, he screams at the 140 decibels. Linsey notices that Shaoyang’s mouth while screaming forms a circle with a radius of 8 inches. What is the difference between the areas of the largest triangle that can be circumscribed by the circle and the smallest triangle that can be inscribed by the circle?

The inradius of the outer triangle is 8. Thus, since it is an equilateral triangle, we know that the side length must be  $16\sqrt{3}$ . For the inner triangle, we know that the circle is a circumcircle. Thus, since the circumradius is 8, the height of the triangle is 12 and the side length is  $8\sqrt{3}$ . The difference in areas is thus  $192\sqrt{3} - 48\sqrt{3} = \boxed{C, 144\sqrt{3}}$ .

13. Holding onto a branch, Shaoyang shouts a question at Linsey: “WHICH OF THE FOLLOWING SIDE LENGTHS COULD NOT FORM A TRIANGLE?” What should Linsey answer to get the question right?

Note that  $25 + 50 = 75$ , which is not greater than 75, so from the Triangle Inequality, the answer is  $\boxed{D, 25, 50, 75}$ .

14. After answering the question correctly, Tim and Linsey make a run for it. While running, the pair spot Miles the Spider Monkey on his web. Miles’s web is in the shape of an octagon with a side length of 8 meters. What is the length of the apothem of the octagon?

The area of the octagon is  $2 \cdot (1 + \sqrt{2}) \cdot 8^2$ . Since this is also equivalent to the semiperimeter, which is  $8 \cdot 8/2 = 32$ , times the apothem, we can divide the area by the semiperimeter. This gives us  $4 \cdot (1 + \sqrt{2}) = \boxed{A, 4 + 4\sqrt{2}}$ .

15. Continuing through the Aisle of Monkeys, the pair spot Nelson the Sloth. Nelson is lying on a branch 16 meters above the ground, and Tim and Linsey are 12 meters from the base of the tree. The tree and the ground make a right angle. Tim bets Linsey that he can web his way straight to the top of the tree (along the hypotenuse) and get to the top before Linsey can run the 12 meters to the tree and climb 16 meters up. If Tim webs at a rate of 3 m/s and Linsey runs and climbs at rates of 5 m/s and 4 m/s respectively, who reaches the top first and how long does it take them, rounded to the nearest tenth?

The length of the hypotenuse is  $\sqrt{12^2 + 16^2} = 20$ . Thus, it takes Tim  $20/3 = 6.67$  seconds to get to the top of the tree. Linsey runs 12 at 5m/s, so she reaches the bottom of the tree in  $12/5 = 2.4$  seconds. Linsey climbs up the tree is  $16/4 = 4$  seconds. Thus, it takes Linsey 6.4 seconds to reach the top, which is faster than Tim.  $\boxed{B, \text{ Linsey, 6.4 seconds}}$ .

16. As soon as both of them reach the top, Heewon the Harpy Eagle swoops in and eats Nelson. Stunned, Tim notices that Heewon’s wings look like two  $13 - 14 - 15$  triangles. What is the total area of Heewon’s wings?

The triangle area is commonly memorized to be 84. It can also easily be found using Heron’s formula. Since there are 2 wings, the total area is  $\boxed{B, 168}$ .

17. Hadriel the Macaw spots Tim still in shock and agrees to help the pair get to the perfect worm if they correctly answer his 3 riddles. Tim, immediately forgetting the death he just witnessed, eagerly accepts. But he needs your help to answer all of them! Riddle 1: My favorite leaf is a triangle  $\triangle ABC$ , where point  $D$  is on  $\overline{AC}$  such that  $\overline{BD}$  bisects  $\angle ABC$ . If  $AB = 6$ ,  $AD = 3$ , and  $DC = 2$ , compute  $BC$ .

From the Angle Bisector Theorem,

$$\frac{6}{AB} = \frac{AB}{BC} = \frac{AD}{DC} = \frac{3}{2} \implies AB = 6 \cdot \frac{2}{3} = \boxed{A, 4}.$$

18. Not bad! Riddle 2: Jason is a fearless explorer who comes across a temple dedicated to the Geometry gods. In the temple, he spots a mural depicting a man and his dad putting a bomb in the sink! The bomb is a triangle  $\triangle ABC$  with side lengths  $AB = 5$ ,  $BC = 6$ , and  $CA = 2$ . Jason wants to add his own touch to the mural so he draws a point  $D$  located on segment  $\overline{BC}$ . If  $BD = 2$ , find the length of  $AD$ .

Note that  $DC = 6 - 2 = 4$ , so from Stewart's Theorem,

$$4 \cdot 2 \cdot 6 + 6 \cdot AD^2 = 5^2 \cdot 4 + 2 \cdot 2^2 \implies 6AD^2 = 60 \implies AD = \boxed{D, \sqrt{10}}.$$

19. Wow, you're almost there! Riddle 3: Two jungle trees (with negligible width), 30 ft and 50 ft tall, are 40 ft apart and perpendicular to the ground. The trees are supported by vines attached from the top of each tree to the bottom of the other. If  $a$  is the height from the ground at the point where the two vines cross, and  $b$  is the horizontal distance from the point where the two vines cross to the taller tree, what is  $a + b$  in feet?

The height of the intersection point of the vines is given by

$$\frac{1}{h} = \frac{1}{30} + \frac{1}{50} = \frac{80}{1500} = \frac{1}{150/8} \implies h = 1508 = 18.75.$$

From similar triangles, the horizontal distance from this point to the taller tree is

$$\frac{18.75 \text{ ft}}{30 \text{ ft}} \cdot 40 \text{ ft} = 25 \text{ ft},$$

$$\text{so } a + b = 18.75 + 25 = \boxed{D, 43.75}.$$

20. Tim successfully answers all 3 riddles correctly, so Hadriel agrees to fly them. While flying, they spot the Jaguars James and Jay having a duel to the death to see who is the better Jaguar. Tim notices that their footprints are in the shape of a regular pentagon with a side length of 11. What is the semiperimeter of James's foot?

The perimeter of the pentagon is  $5 \cdot 11 = 55$ , so the semiperimeter is  $\boxed{E, \frac{55}{2}}$ .

21. While flying, a breeze comes by that pushes Linsey off! After reaching the point  $(9, 17)$ , Hadriel nose dives towards Linsey on the line  $y = \frac{4}{3}x + 5$  while traveling at a rate of 5 units per second. If Linsey starts at the point  $(3, 17)$  and is falling at a rate of 4 units per second in the negative  $y$ -direction and the ground is represented as the  $x$ -axis, will Hadriel catch her, and if so, at what coordinates?

Note that their paths intersect where the lines  $x = 3$  and  $y = \frac{4}{3}x + 5$  do, which is at  $(3, 9)$ . This point is 10 units away from Hadriel's starting point and 8 units away from Linsey starting point. It takes Hadriel  $10/5 = 2$  seconds and Linsey  $8/4 = 2$  seconds to reach this point, so the answer is  $\boxed{B, \text{Yes}, (3, 7)}$ .

22. Luckily, Tim uses his webbing to catch Linsey. Hadriel drops Tim and Linsey at the top of a tree. The tree can be represented as a cylinder with radius 2 meters and height of 69 meters, topped with a cone of radius 8 meters and height of 42 meters. Find the volume of the tree.

The volume of the cylinder is  $2^2 \cdot 69 \cdot \pi = 276\pi$ , and the volume of the cone is  $\frac{1}{3} \cdot 8^2 \cdot 42 \cdot \pi = 64 \cdot 14 \cdot \pi = 896\pi$ , so the volume of the tree is  $276\pi + 896\pi = \boxed{D, 1172\pi}$ .

23. While continuing on their journey, Linsey wonders whether Tim is worthy to be the Tarantula King. As a test, Linsey asks Tim what the external tangent of 2 circles with centers 25 units apart and radii of 11 and 4 units is. To prove that he is the true Tarantula King, what should Tim answer?

We can construct the right triangle with hypotenuse connecting the centers of the circles and a leg parallel to the external tangent. This triangle has hypotenuse 25 and one leg length  $11 - 4 = 7$ . Then the length of the other leg, which is equal to the length of the external tangent, is  $\sqrt{25^2 - 7^2} = \boxed{E, 24}$ .

24. Tim gets it right! However, Linsey is still skeptical, and asks another question: What is the length of the internal tangent of the two circles in last question? Help Tim answer this correctly.

Connecting the centers of the circles to the tangency points of the internal tangent, we can “shift” segments to get a right triangle with hypotenuse 25 and leg  $11 + 4 = 15$ . Then the length of the internal tangent is the length of the other leg, which is  $\sqrt{25^2 - 15^2} = \boxed{C, 20}$ .

25. Tim and Linsey are almost at the end of their journey! They run into Arib the Indian Anaconda, who is not only the guardian of the perfect meal, but the longest creature in the world at a whopping 350 meters. If the average person is 175 cm, how many times as long as the average person is Arib?

One meter is 100 centimeters, so the answer is  $100 \cdot \frac{350}{175} = 100 \cdot 2 = \boxed{A, 200}$ .

26. Arib gives Tim the location of the perfect meal, a majestic, delectable, whimsical worm. If it is at the point  $(80, 50)$  on the Cartesian plane and Arib and Tim are located at the point  $(20, 25)$ , what is the shortest distance that Tim must travel to get to the worm?

From the distance formula, the answer is  $\sqrt{(80 - 20)^2 + (50 - 25)^2} = \sqrt{60^2 + 25^2} = \boxed{B, 65}$ .

27. After a long journey, Tim reaches and eats the perfect worm. The worm is in the shape of a cylinder of radius 2 and height 8 with hemispheres of radii 2 on the end of the cylinder. What is the surface area of the worm?

The surface area of the hemispherical portions combines to the surface area of one sphere with radius 2, which is  $4 \cdot 2^2 \cdot \pi = 16\pi$ . The surface area of the cylindrical portion of the worm is  $2 \cdot 2 \cdot 8 \cdot \pi = 32\pi$  so the answer is  $16\pi + 32\pi = \boxed{A, 48\pi}$ .

28. It's a trap! While eating the worm, Tim is trapped in a cage by Mr. Hallett, who Arib had given a heads up to. If the cage is in the shape of a  $4 \times 8 \times 11$  box without a top (assume the top is a  $4 \times 8$  side) and is made up of graphene, how many square units of graphene did Mr. Hallett use to make the cage? (Assume the box's sides are 2 dimensional and that it is empty on the inside.)

The surface area of the cage excluding the top face is

$$4 \cdot 8 + 2(8 \cdot 11 + 11 \cdot 4) = 32 + 264 = \boxed{E, 296}.$$

29. In order to save Tim, Linsey carves her log (from question 2) into a key. If the key is in the shape of a rectangular prism with dimensions  $2 \text{ cm} \times 2 \text{ cm} \times 6 \text{ cm}$ , can Linsey carve her log to fit the dimensions of the key, and if so, how much does she have to shave off (in  $\text{cm}^3$ )?

The log has radius 2 cm and height 8 cm. A square with side length 2 cm can fit in a circle with radius 2 cm, and 6 cm is less than 8 cm, so the log can be carved. The volume of the log is  $32\pi$ , and the volume of the key is  $2 \cdot 2 \cdot 6 = 24$ , so the answer is  $\boxed{B, \text{ Yes, } 32\pi - 24}$ .

30. Linsey is able to get Tim out and the pair escape! With his mission complete, Tim decides to head home. Before leaving, he leaves a marking on the tree closest to where he found the perfect worm. The marking is a triangle with side lengths of 6, 8, and 10. What is the area of the triangle?

This is a right triangle, so the area is  $\frac{1}{2} \cdot 6 \cdot 8 = \boxed{B, 24}$ .